General Wave funcⁿ,

$$\psi(x) = A \sin kx + B \cos kx$$

$$\psi(x) = A \sin \frac{n\pi x}{a} + B \cos \frac{n\pi x}{a}$$

for odd values of $n$

$$\psi(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right) \quad , n = 1, 3, 5 \ldots$$

for even value of $n$,

$$= \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad , n = 2, 4, 6 \ldots$$

$$K^2 = \frac{2mE}{\hbar^2} \Rightarrow \boxed{E = \frac{n^2\pi^2\hbar^2}{2ma^2}}$$

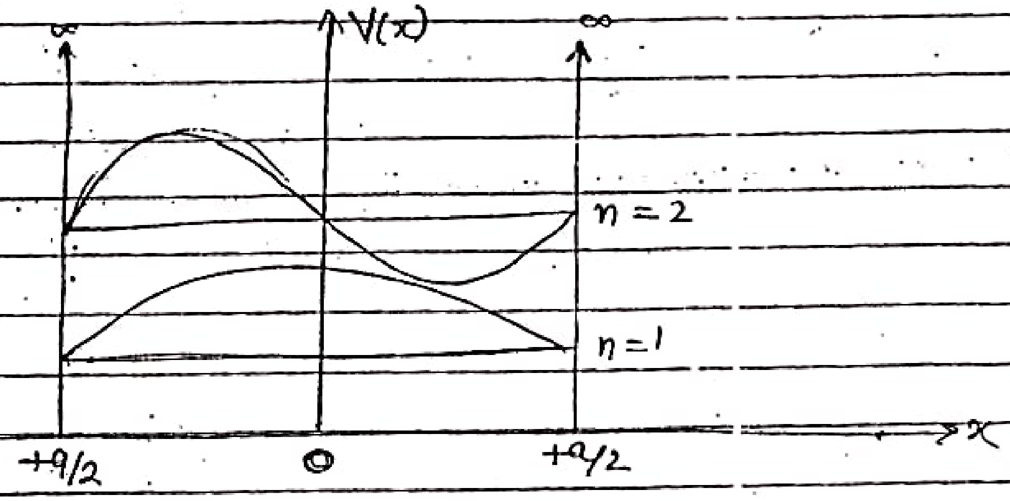
n th state wave funcⁿ for symmetric potⁿ well i.e.

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left[\frac{n\pi}{a}(x + a/2)\right] \quad ; n = 1, 2, 3, 4 \ldots$$

Wave funcⁿ of unsymmetric potⁿ well

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

Put $x = a/2 + x$, we get w.funcⁿ for symmetric P.W.




Expectation value of $x$, $\langle x \rangle = \int_{-a/2}^{a/2} \psi^* x \psi \, dx$

$+a/2$ to $\infty \rightarrow \psi = 0$ $-a/2$ to $a/2$.

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$$\langle x \rangle = \int_{-a/2}^{a/2} \left(\sqrt{\frac{2}{a}}\right)^2 \sin^2 \frac{n\pi}{a}\left(x+\frac{a}{2}\right) \cdot x \, dx \quad \{\text{odd fu}^n\}$$

$$\langle x \rangle = 0$$

$$\langle p \rangle = \int_{-a/2}^{+a/2} \frac{2}{a} \sin\left[\frac{n\pi}{a}\left(x+\frac{a}{2}\right)\right]\left(-i\hbar\frac{\partial}{\partial x}\right) \sin\left[\frac{n\pi}{a}\left(x+\frac{a}{2}\right)\right] dx = 0$$

Problem :- The normalise wave fu$^n$ of a particle in 1 Dim box extending from 0 to L is

$$\psi(x) = \begin{cases} \sqrt{\frac{12}{L^3}}\, x & \text{for } 0 < x < \frac{L}{2} \\ \sqrt{\frac{12}{L^3}}\, L-x & \text{for } \frac{L}{2} < x < L \\ 0 & \text{otherwise} \end{cases}$$

The prob. of finding the particle in ground state

(a) $\frac{98}{L^4}$ (b) $\frac{96}{\pi^4}$ (c) $\frac{96}{\pi^3}$ (d) $\frac{48}{\pi^3}$

$$|\psi|^2 = \int_0^{L/2} \sqrt{\frac{12}{L^3}}\, x \cdot \sqrt{\frac{12}{L^3}}\, x \, dx + \int_{L/2}^{L} \sqrt{\frac{12}{L^3}}(L-x)\sqrt{\frac{12}{L^3}}(L-x)\, dx$$

$$= \int_0^{L/2} \left(\frac{12}{L^3}\right) x^2 dx + \left(\frac{12}{L^3}\right)\int_{L/2}^{L} (L-x)^2 dx$$

$$= \frac{12}{L^3}\left(\frac{x^3}{3}\right)_0^{L/3} + \left(\frac{12}{L^3}\right)\left[L^2 x + \frac{x^3}{3} - 2L\frac{x^2}{2}\right]_{L/2}^{L}$$

$$= \frac{1}{3}\frac{12}{L^3}\left[\frac{L^3}{27}\right] + \frac{12}{L^3}\left[L^3 + \frac{L^3}{3} - L^3 - \frac{L^3}{2} + \frac{L^3}{8\times 3} - \frac{L^3}{4}\right]$$

//

Wave fu$^n$ for the particle in 1-D box,

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

for $n=1$, $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$

Prob. of finding the particle in ground state is $\rightarrow |c_1|^2$

$$c_1 = \int_{-\infty}^{+\infty} \phi_1^*(x)\,\psi\,dx$$

$$c_1 = \int_0^{L/2} \phi_1^*(x)\psi(x)\,dx + \int_{L/2}^{L} \phi_1^*(x)\psi(x)\,dx +$$

$$c_1 = \int_0^{L/2} \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}\cdot\sqrt{\frac{12}{L^3}}\,x\,dx + \int_{L/2}^{L} \sqrt{\frac{2}{L}}\sin\frac{\pi x}{2}\sqrt{\frac{12}{L^3}}(L-x$$

$$c_1 = \sqrt{\frac{24}{L^4}}\left[\int_0^{L/2} x\sin\frac{\pi x}{L}\,dx + \int_{L/2}^{L}(L-x)\sin\frac{\pi x}{L}\,dx\right]$$

$$= \sqrt{\frac{24}{L^4}}\left\{\left[-x\cos\frac{\pi x}{L}\left(\frac{L}{\pi}\right) + \frac{\sin\frac{\pi x}{L}}{\left(\frac{\pi}{L}\right)^2}\right]_0^{L/2} + \left[-(L-x)\cos\frac{\pi x}{L} - \frac{\sin\pi x/L}{(\pi/L)^2}\right]_{L/2}^{L}\right\}$$

$$= \frac{\sqrt{24}}{L^2}\left\{\left[0+\frac{L^2}{\pi^2}\right] + \left[0+\frac{L^2}{\pi^2}\right]\right\}$$

$$= \frac{\sqrt{24}}{L^2}\left[\frac{2L^2}{\pi^2}\right]$$

$$c_1 = \frac{2\sqrt{24}}{\pi^2} = \frac{\sqrt{96}}{\pi^2}$$

Prob. of finding the particle in ground stat is $= |c_1|^2$

$$|c_1|^2 = \frac{96}{\pi^4}$$ Ans (b)